A fast solution for the optimal location of the DC combiner box in a PV array

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ABSTRACT

In PV power systems the choice of an appropriate location for the installation of the PV array box (or DC combiner box) is an important undertaking. It is essential that the box be placed so that the amount of DC cabling is minimized in order to not only save cable costs but also reduce voltage losses. This paper presents a fast solution to this problem, based on a mathematical model for the minisum location of the combiner according to the Manhattan metric between the PV array and the DC combiner box. The target function and its optimal solution (i.e. the most economical amount of cabling) for this particular model were obtained, and the optimality of the solution proved by contradiction. The application of this model is illustrated by means of two typical examples, involving an odd and an even number of strings in a PV array. The proposed model is efficient and easy to apply, and as such should be of interest to PV engineers and designers.

Introduction

One of the major classical problems in operations research is the decision of optimal location, a subject of great importance in applications such as production, logistics, life in general and even military affairs. The problem has recently attracted a lot of interest in the fields of operational research, engineering science and management, as well as among computer scientists: indeed, many new problems with a practical application background are emerging as the study of the location problem becomes more profound. Among all the location research fields, facility location is one of the most important – the range of facilities may include offices, warehouses, batch plants, maintenance facilities, labour force residences and fabrication shops. The approaches and solutions to the problem of facility location mainly rely on operational research, topology, management and so on [1,2].

The choice of PV array combiner box location discussed in this paper belongs to the facility location problem; this is an important element of the whole PV system design because the decision will significantly affect cost and efficiency, as well as other aspects of the PV system. Since the cost of DC cabling is often directly proportional to the distance between the PV strings and the combiner box, the location of the combiner should be sufficiently optimal to minimize both the cost of the DC cables and the power losses in the cables, which in turn will enhance system efficiency. This paper proposes a solution to the problem of combiner location using a mathematical optimization approach based on the Manhattan metric algorithm (distance between two facilities measured along the x and y axes) [1,2]. The mathematical model for the minisum location of the combiner is established by taking the real arbitrary PV system array into consideration, and then obtaining the target function and its optimal solution. Examples of the application of the technique are presented for both an odd and an even number of PV strings.

“The location of the combiner should be sufficiently optimal to minimize both the cost of the DC cables and the power losses in the cables”

Single-combiner location model

There is generally one combiner box in a PV array, so this is a single-facility location problem. The strings of PV modules are connected in parallel to the combiner through DC cables running along the cable trays, which are often set vertically or aligned in parallel with each other.

Assumption

The objective area for the location is continuous, and any point is a candidate for the optimal solution. In addition, for the distance between two points, rectangular distance is used, which is approximately equal to the real distance.

Problem definition

Considering a coordinate system (x, y), there exists a boundary of a planar region with given coordinate values of a number of existing points. It is desired to find the point in the coordinate system that minimizes the sum of the rectangular distances to all the existing points.

Mathematical model

Suppose there are n strings located in the PV array. A coordinate system (x, y) is established (the coordinate origin is at any position) so that the coordinates of each positive or negative output node of the strings can be measured; there are therefore $2n$ points in total. Here, suppose that (x_i, y_i) are the coordinates of the point i, while the combiner box is located at (x_p, y_p). The rectilinear, or Manhattan, distance between the point (x_i, y_i) and the combiner (x_p, y_p) is then defined as $|x_p - x_i| + |y_p - y_i|$, for $i = 1, 2, \ldots 2n$. The objective function of the combiner location can then be expressed as

$$\text{Min } L(x_p, y_p) = \sum_{i=1}^{2n} |x_p - x_i| + |y_p - y_i| \qquad (1)$$

where $x_p \in [x_1, x_{2n}]$ and $y_p \in [y_1, y_{2n}]$.

Solution of the objective function

It is clear that Equation 1 is multinomial, i.e. it comprises two independent components: the distances along the x and y axes, which add to the total distance independently of each other. Hence, the objective function can also be expressed as:

$$\text{Min } L(x_p, y_p) = \sum_{i=1}^{2n} |x_p - x_i| + \sum_{i=1}^{2n} |y_p - y_i| \qquad (2)$$

that is

$$\begin{aligned} L(x_p, y_p) &= \left(|x_p - x_1| + \cdots + |x_p - x_{2n}|\right) \\ &\quad + \left(|y_p - y_1| + \cdots + |y_p - y_{2n}|\right) \\ &= L(x_p) + L(y_p) \end{aligned} \qquad (3)$$

Hence,

$$L(x_p) = |x_p - x_1| + \cdots + |x_p - x_{2n}| \qquad (4)$$

$$L(y_p) = |y_p - y_1| + \cdots + |y_p - y_{2n}| \qquad (5)$$

where $x_p \in [x_1, x_{2n}]$ and $y_p \in [y_1, y_{2n}]$.

It is clear that this task can be solved separately for the x and y coordinates and then the results merged in order to obtain the rectangle of minimum distance points. Let x_p^* and y_p^* denote the optimal answers for

$$\text{Min} \sum_{i=1}^{2n} |x_p - x_i| \text{ and Min} \sum_{i=1}^{2n} |y_p - y_i|$$

respectively. Thus, the coordinates of the optimal point of the final solution are (x_p^*, y_p^*). A detailed induction of the optimal solution of $L(x_p)$ is given next.

Suppose we have $2n$ points ordered by x, namely $x_1, x_2, \ldots x_{2n}$, as shown in Fig. 1. It should be noted that the values of each of these may or may not be the same, which mainly depends on the position of the positive or negative outputs of the PV strings.

From Fig. 1 it is clear that x_p^* may be any of the points $x_1, x_2, \ldots x_{2n}$, or may lie in the intervals $[x_1, x_2]$, $[x_2, x_3]$, ... $[x_{2n-1}, x_{2n}]$, so the solution has $(4n-1)$ possibilities in total. Consider x_p^* to be in the interval $[x_i, x_{i+1}]$; by removing the absolute values, Equation 4 may also be written as

$$\begin{aligned} L(x_p) &= (x_p - x_1) + (x_p - x_2) \cdots + (x_p - x_i) + (x_{i+1} - x_p) + \cdots + (x_{2n} - x_p) \\ &= i * x_p - (x_1 + \cdots + x_i) + [-(2n-i)] * x_p + (x_{i+1} + \cdots + x_{2n}) \\ &= (2i - 2n) * x_p + [(x_{i+1} + \cdots + x_{2n}) - (x_1 + \cdots + x_i)] \end{aligned} \quad (6)$$

Therefore, $L(x_p)$ in any segment $[x_i, x_{i+1}]$ is given by Equation 6, where $x_p \in [x_i, x_{i+1}]$, for $i = 1, 2, \ldots 2n-1$. It is evident that Equation 6 is a piecewise function over the entire segment $[x_1, x_{2n}]$.

Similarly, $L(y_p)$ in any interval $[y_i, y_{i+1}]$ can be written as

$$L(y_p) = (2i - 2n) * y_p + [(y_{i+1} + \cdots + y_{2n}) - (y_1 + \cdots + y_i)] \quad (7)$$

where $y_p \in [y_i, y_{i+1}]$, for $i = 1, 2, \ldots 2n-1$. It is also easy to see that Equation 7 is a piecewise linear function over the entire interval $[y_1, y_{2n}]$.

As can be seen from Equation 6, $L(x_p)$ is a linear function of x_p: when $i > n$, the slope of the piecewise function is positive in any interval, whereas when $i < n$, the slope is negative. When $(2i - 2n)$ is equal to zero, the slope of the piecewise function is zero. According to the properties of a monotonic function, over the entire interval $[x_1, x_{2n}]$ the value of Min $L(x_p)$ is equal to $L(x_p)|_{i=n}$, i.e.

$$\begin{aligned} L(x_p)|_{i=n} &= [(x_{i+1} + \cdots + x_{2n}) - (x_1 + \cdots + x_i)]|_{i=n} \\ &= (x_{n+1} + \cdots + x_{2n}) - (x_1 + \cdots + x_n) \end{aligned} \quad (8)$$

so the resulting optimal solution x_p^* lies in the interval $[x_n, x_{n+1}]$.

Similarly, the optimal solution of Min $L(y_p)$ is $[y_n, y_{n+1}]$. Hence, the optimal solution of Min $L(x_p, y_p)$ is

$$\{(x_p, y_p) | x_p = [x_n, x_{n+1}], y_p = [y_n, y_{n+1}]\} \quad (9)$$

> "The optimal solution x_p^* lies in the interval $[x_n, x_{n+1}]$."

Equation 9 is a fundamental solution of the functions in Equations 6 and 7. In particular, the values of x_n and x_{n+1} may or may not be the same (the same applies to y_n and y_{n+1}), depending on the numbers and positions of the PV strings, so the global optimal solution of the combiner location problem can be divided into the following four cases:

Case A: $[x_n, y_n]$ or $[x_{n+1}, y_{n+1}]$,
where $x_n = x_{n+1}$, $y_n = y_{n+1}$

Case B: $\{(x_p, y_p) | x_p = x_n, y_p = [y_n, y_{n+1}]\}$,
where $x_n = x_{n+1}$, $y_n \neq y_{n+1}$

Case C: $\{(x_p, y_p) | x_p = [x_n, x_{n+1}], y_p = y_n\}$,
where $x_n \neq x_{n+1}$, $y_n = y_{n+1}$

Case D: $\{(x_p, y_p) | x_p = [x_n, x_{n+1}], y_p = [y_n, y_{n+1}]\}$,
where $x_n \neq x_{n+1}$, $y_n \neq y_{n+1}$

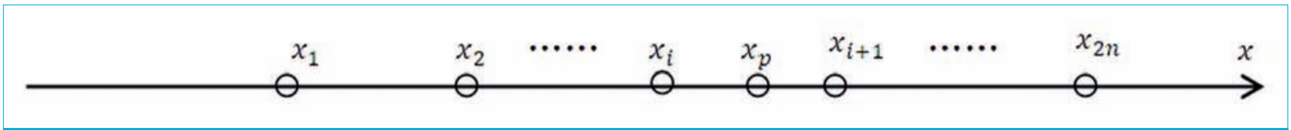

Figure 1. $2n$ points ordered by x on a line.

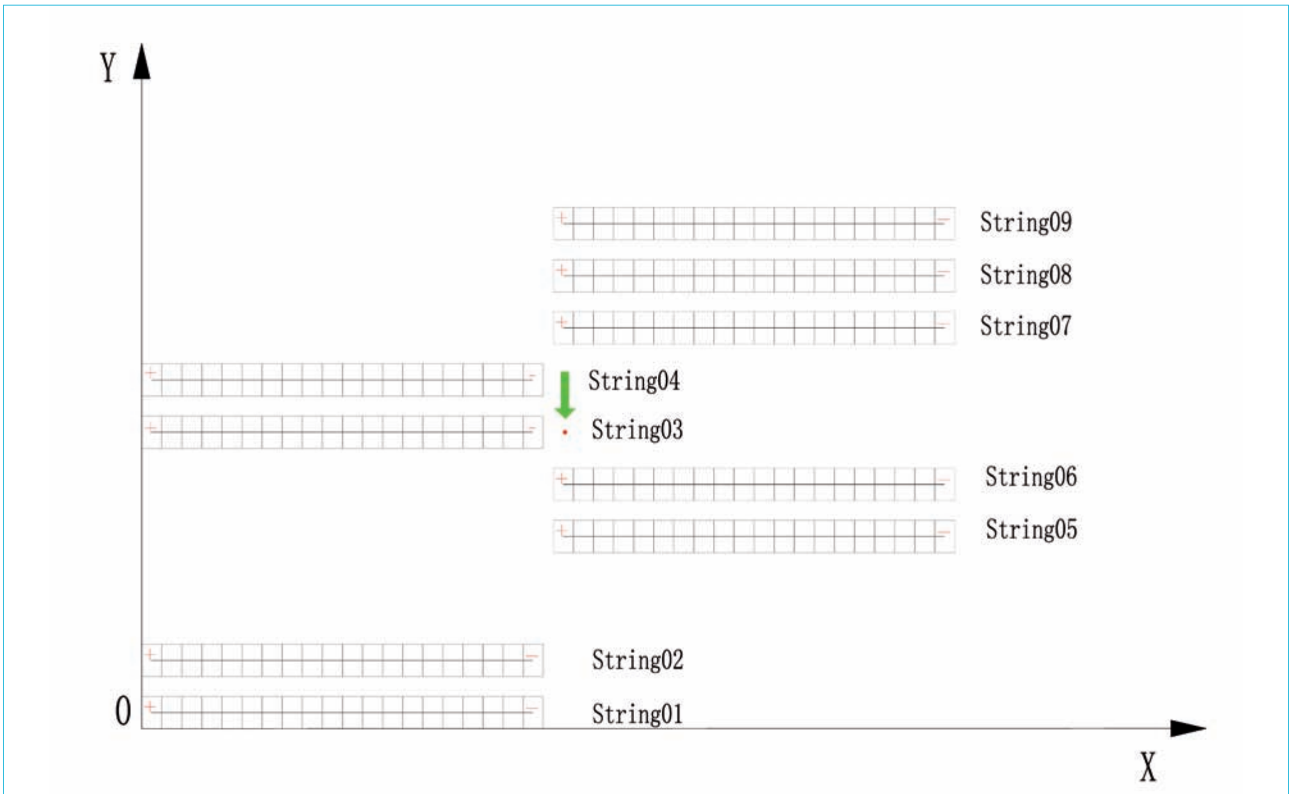

Figure 2. Case B: Optimal location of the combiner for an odd number of PV strings.

As seen above, case A represents a single point, and cases B and C denote a union of a whole segment. Case D, on the other hand, denotes a rectangular region, which means that more than one point may exist for the solution.

Verification of the optimal solution

The proof that $[x_n, x_{n+1}]$ is the optimal solution of Min $L(x_p)$ will be established by contraposition. Suppose the optimal solution lies in the interval $[x_m, x_{m+1}]$, for $i = m$ and where $m < n$ or $m > n$, so that $x_m \geq x_n$ or $x_m \leq x_n$. Thus, $L(x_p)|_{i=n} - L(x_p)|_{i=m} > 0$, where $L(x_p)|_{i=m} = (2m - 2n) * x_p + [(x_{m+1} + \dots + x_{2n}) - (x_1 + \dots + x_m)]$.

Proof

$$\begin{aligned} L(x_p)_{|i=n} - L(x_p)_{|i=m} &= (x_{n+1} + \cdots + x_{2n}) \\ &\quad - (x_1 + \cdots x_m + x_{m+1} + \cdots x_n) - (2m - 2n) * x_p \\ &\quad - [(x_{m+1} + \cdots + x_{2n}) - (x_1 + \cdots + x_m)] \\ &= (x_{n+1} + \cdots + x_{2n}) - (x_{m+1} + \cdots x_n) + 2(n - m) * x_p \\ &\quad - (x_{m+1} + \cdots + x_n) - (x_{n+1} + \cdots + x_{2n}) \\ &= 2(n - m) * x_p - 2(x_{m+1} + \cdots + x_n) \end{aligned} \tag{10}$$

where $x_p \in [x_m, x_{m+1}]$.

- When n < m, $L(x_p)|_{i=n} - L(x_p)|_{i=m} < 0$, so $L(x_p)|_{i=n} < L(x_p)|_{i=m}$ (11)
- When n > m, $L(x_p)|_{i=n} - L(x_p)|_{i=m} = 2[(n - m)\, x_p - (x_{m+1} + \dots + x_n)]$ (12)

Since $x_p \in [x_m, x_{m+1}]$, this means $x_m \leq x_p \leq x_{m+1} \leq x_{m+2} \dots \leq x_n$. From Equations 11 and 12, $L(x_p)|_{i=n} - L(x_p)|_{i=m} \leq 0$, which is a contradiction. Thus, when $i = n$, it is proved that $[x_n, x_{n+1}]$ is the optimal solution of Min $L(x_p)$. Similarly, it can be proved that when $i = n$, then $[y_n, y_{n+1}]$ is the optimal solution of Min $L(y_p)$.

Analysis of actual cases

From a practical point of view, the number of PV strings may be odd or even, or their arrangement may be irregular. Given the values of the located points in a PV array, it is a simple task to obtain the optimal location for the combiner on the basis of the conclusion of the previous section. Two examples corresponding to the cases A and B discussed earlier will be analyzed.

String number	Coordinate of positive output of string		Coordinate of negative output of string	
	X+ [m]	Y+ [m]	X- [m]	Y- [m]
1	0.492	0.808	19.992	0.808
2	0.492	3.408	19.992	3.408
3	0.492	14.789	19.992	14.789
4	0.492	17.389	19.992	17.389
5	20.5	9.589	40.492	9.589
6	20.5	12.189	40.492	12.189
7	20.5	19.989	40.492	19.989
8	20.5	22.589	40.492	22.589
9	20.5	25.189	40.492	25.189

Table 1. Case A: Coordinate value of the output of each string.

Number	x_i [m]	y_i [m]	Number	x_i [m]	y_i [m]
1	0.492	0.808	10	20.5	14.789
2	0.492	0.808	11	20.5	17.389
3	0.492	3.408	12	20.5	17.389
4	0.492	3.408	13	20.5	19.989
5	19.992	9.589	14	40.492	19.989
6	19.992	9.589	15	40.492	22.589
7	19.992	12.189	16	40.492	22.589
8	19.992	12.189	17	40.492	25.189
9	20.5	14.789	18	40.492	25.189

Table 2. Case A: Coordinate value of the output of each string, in order along the x and y axes.

Example 1: case A

Consider nine strings in a PV array and eighteen given points whose locations are as shown in Fig. 2. The numerical values of the coordinates are shown in Table 1. The origin of the coordinate system is the left corner of string number one. The coordinate value of the output of each string, in order along the x and y axes, is shown in Table 2.

According to the values provided in Tables 1 and 2, the resulting optimal solution of the combiner location problem is the point where $x_p^* \in [x_9, x_{10}]$ and $y_p^* \in [y_9, y_{10}]$. However, $x_9 = x_{10}$ and $y_9 = y_{10}$, so it follows that the optimal solution is just the point [20.5, 14.789], as shown by the red dot in Fig. 2.

Example 2: case B

Now consider ten strings in a PV array and twenty given points whose locations are as shown in Fig. 3. The numerical values of the coordinates are presented in Tables 3 and 4.

The optimal location of the combiner is the point satisfying $x_p^* \in [x_{10}, x_{11}]$ and $y_p^* \in [y_{10}, y_{11}]$, where $x_{10} = x_{11}$ and $y_{10} \neq y_{11}$. The optimal solution is therefore a segment defined by $x_p^* = 20.5$ and $y_p^* \in [14.789, 17.389]$: this is the segment between the negative outputs of the third and fourth strings of the PV array, as shown by the red line in Fig. 3.

"The solution of the location problem can be quickly determined manually."

Conclusion

This paper has described a technique for determining the optimum location of the combiner by using a mathematical optimization function based on the Manhattan metric algorithm. The objectives of the optimization problem were to minimize both the investment in overall material cost and the system voltage losses in the cables. It was shown that the Manhattan-based optimal solution of the combiner location problem can be drawn from a finite set of candidate points (positive or negative outputs of PV strings), all of which are easy to determine. The technique was illustrated by means of two examples.

From a practical point of view, although the solution of the location problem presented in this paper cannot be obtained using computer software, it can be quickly determined manually. Moreover, the optimization model can be implemented for any size of PV array. However, if the resulting optimal location for the installation of the combiner is not feasible, the next best option is to choose the most practical placement closest to the optimal position.

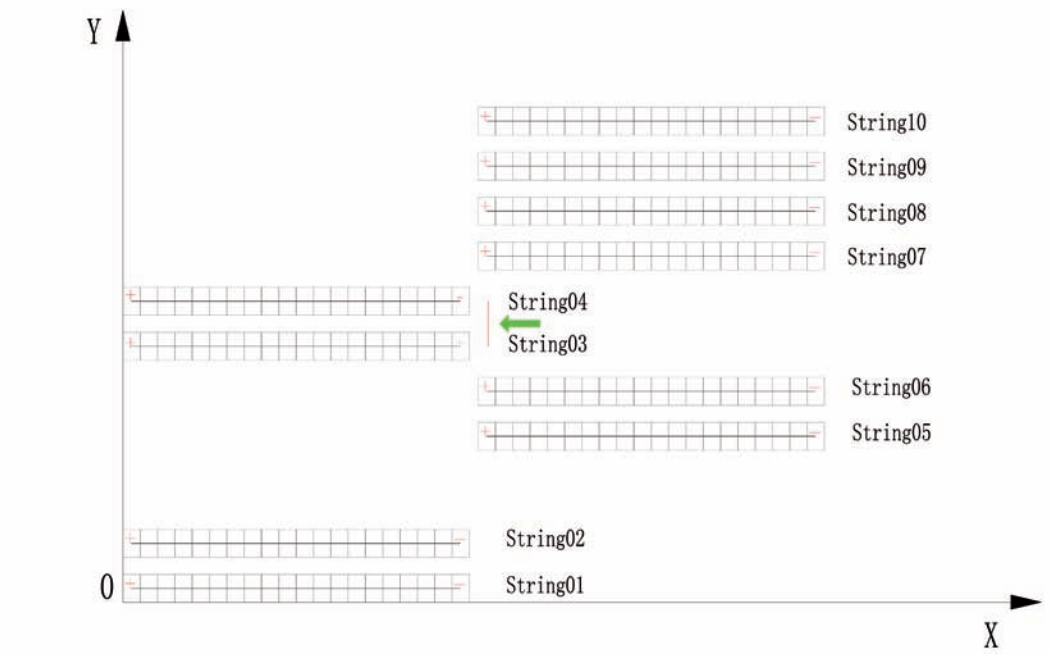

Figure 3. Optimal location of the combiner for an even number of PV strings.

String number	Coordinate of positive output of string		Coordinate of negative output of string	
	X+ [m]	Y+ [m]	X- [m]	Y- [m]
1	0.492	0.808	19.992	0.808
2	0.492	3.408	19.992	3.408
3	0.492	14.789	19.992	14.789
4	0.492	17.389	19.992	17.389
5	20.5	9.589	40.492	9.589
6	20.5	12.189	40.492	12.189
7	20.5	19.989	40.492	19.989
8	20.5	22.589	40.492	22.589
9	20.5	25.189	40.492	25.189
10	20.5	22.789	40.492	22.789

Table 3. Case B: Coordinate value of the output of each string.

Number	x_i [m]	y_i [m]	Number	x_i [m]	y_i [m]
1	0.492	0.808	11	20.5	17.389
2	0.492	0.808	12	20.5	17.389
3	0.492	3.408	13	20.5	19.989
4	0.492	3.408	14	20.5	19.989
5	19.992	9.589	15	40.492	22.589
6	19.992	9.589	16	40.492	22.589
7	19.992	12.189	17	40.492	25.189
8	19.992	12.189	18	40.492	25.189
9	20.5	14.789	19	40.492	27.789
10	20.5	14.789	20	40.492	27.789

Table 4. Case B: Coordinate value of the output of each string, in order along the x and y axes.

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